Sesión Especial 23

Métodos numéricos para la resolución de problemas no lineales

Organizadores

- Sergio Amat (Universidad Politécnica de Cartagena)
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Descripción

Es bien conocido que la resolución de diferentes tipos de problemas se puede modelizar a partir de una ecuación no lineal, de manera que la resolución de ecuaciones no lineales es uno de los problemas matemáticos que aparecen con mayor frecuencia en las diferentes disciplinas científicas. Y los métodos iterativos son la pieza clave en la resolución de este tipo de ecuaciones.

La resolución de ecuaciones no lineales mediante métodos iterativos es una línea de investigación de interés, tanto desde el punto de vista de la matemática pura como de la matemática aplicada.

Esta sesión especial se sitúa en el marco general de la aplicación de métodos iterativos para la resolución de problemas no lineales, centrando nuestro interés en el tratamiento numérico de la resolución de diversos tipos de problemas en los que aparecen ecuaciones no lineales de diversa índole: ecuaciones escalares, sistemas de ecuaciones, ecuaciones matriciales, ecuaciones integrales, problemas de valores en la frontera, ecuaciones en derivadas parciales, problemas de optimización, etc. Y los objetivos de esta sesión se centran entonces en mostrar los últimos avances llevados a cabo en este campo.

Programa

Lunes, 4 de febrero (mañana)

11:30 – 12:00	Francisco I. Chicharro (Universidad Internacional de la Rioja) *High-order families of iterative methods for solving nonlinear systems*
12:00 – 12:30	Alicia Cordero (Universitat Politècnica de València) *Modified Potra-Pták multi-step schemes with accelerated order of convergence for solving systems of nonlinear equations*
12:30 – 13:00	Neus Garrido (Universitat Politècnica de València) *Convergence and stability in a new family of iterative methods for solving nonlinear systems of equations*

13:00 – 13:30	Á. Alberto Magreñán (Universidad de La Rioja) *Ball convergence for a novel fourth order method for solving systems of equations*

Lunes, 4 de febrero (tarde)

17:00 – 17:30	José Manuel Gutiérrez (Universidad de La Rioja) *El plano de parámetros del método de Newton aplicado a polinomios de grado cuatro con una raíz doble*
17:30 – 18:00	Sergio Amat (Universidad Politécnica de Cartagena) *Iterative methods for implicit Runge-Kutta schemes*
18:00 – 18:30	Natalia Romero (Universidad de La Rioja) *On an efficient third order iterative scheme for solving a quadratic matrix equation*
18:30 – 19:00	Juan R. Torregrosa (Universitat Politècnica de València) *Iterative Methods for Approximating Generalized Inverse* $A^{(2)}_{T,S}$ *of a Complex Matrix* A

Martes, 5 de febrero (mañana)

11:30 – 12:00	Miguel Ángel Hernández (Universidad de La Rioja) *Majorizing sequences and iterative processes*
12:00 – 12:30	Eulalia Martínez (Universitat Politècnica de València) *Serial and parallel iterative splitting methods: algorithms and applications*
12:30 – 13:00	Jean-Claude Yakoubsohn (Université Paul Sabatier) *Nuevos resultados para la aproximación rápida y certificada de la descomposición en valores singulares*

Iterative methods for implicit Runge-Kutta schemes

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Abstract. In order to approximate stiff ordinary differential equation systems, the Implicit Runge-Kutta methods emerge as good candidates due their stability properties and high orders of accuracy. Of course, we need to approximate their nonlinear stage equations. To reduce the computational cost, several approximate Newton algorithms were developed. In this paper, a new theoretical analysis for some (efficient) of these algorithms is presented.

Referencias

[1] D. Xie, An improved approximate Newton method for implicit Runge-Kutta formulas. Journal of Computational and Applied Mathematics 235 (2011) 5249–5258.

Joint work with Sonia Busquier, Ioannis K. Argyros, Juan Ruíz.
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High-order families of iterative methods for solving nonlinear systems

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Abstract. Over the last few decades, there has been a wide interest in the design of iterative methods for solving both equations and systems of equations that present nonlinearities. Finding a solution of nonlinear systems $F(x) = 0$ is a task that appear frequently in many problems of Science and Engineering and the iterative methods are good candidates for approximating these solutions. In this paper, a generalized high-order family for solving nonlinear systems is presented. This class has four steps and two weight functions on its iterative expression. It needs four functional evaluations and one evaluation of the Jacobian matrix. Previously to its design, it is proposed a fourth-order family with only a weight function, which is the basis for construct our class. Under some conditions of the weight functions, we prove that all the members of the family have order of convergence nine. We also study the computational effort of several members of the proposed family, doing a comparative analysis with the efficiency of other known schemes. The performance of different elements of the family is studied for solving several test systems of nonlinear equations and for Fisher's problem, showing the good features of the proposed schemes.

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Modified Potra-Pták multi-step schemes with accelerated order of convergence for solving systems of nonlinear equations

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Abstract. Finding solutions of nonlinear systems $F(x) = 0$ is an important problem in several branches of Science and Engineering. As to find the exact solution is not possible in many cases, the iterative methods are used for approximating this solution. In this paper, based on Potra-Pták method [1] we develop a three-step scheme. The first two steps are those of Potra-Pták method whereas the third one is a weighted Newton-step. From standard conditions we prove that the proposed scheme has order of convergence six. Furthermore, we generalize the mentioned scheme to derive a family of multi-step iterative methods with order of convergence $2r + 6$, $r = 0, 1, 2, \ldots$ The sixth order method is the special case of this multi-step family for $r = 0$. We also study the computational efficiency of the proposed schemes, doing a comparative analysis with the efficiency of other known methods. Different numerical tests, containing academical functions and systems resulting from the discretization of boundary problems, are introduced to confirm the theoretical results and to show the efficiency and reliability of the proposed schemes.

Referencias

[1] F. A. Potra, V. Pták. Nondiscrete induction and iterative processes. Pitman Publishing, Boston, 1984.

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Convergence and stability in a new family of iterative methods for solving nonlinear systems of equations

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Abstract. Iterative methods are commonly used in many applied problems where the solution of a nonlinear system of equations is required. In this work, the following iterative family for solving nonlinear systems of equations, called G family, is proposed:

$$\begin{aligned} y^{(k)} &= x^{(k)} - \left[F'(x^{(k)})\right]^{-1} F(x^{(k)}), \\ x^{(k+1)} &= x^{(k)} - G(\eta_k)\left[F'(x^{(k)})\right]^{-1} F(x^{(k)}), \qquad k = 0, 1, 2, \ldots, \end{aligned}$$

where, $G : \mathbb{R}^{n\times n} \longrightarrow \mathbb{R}^{n\times n}$ is a matrix weight function with variable $\eta_k = I - \left[F'(x^{(k)})\right]^{-1}\left[y^{(k)}, x^{(k)}; F\right]$. The G family is designed using a composition-type of Newton's method and a weight function procedure. These techniques make possible to achieve higher order of convergence, as family G is at least fourth-order convergent under certain conditions for the function G. The real multidimensional dynamical analysis of particular subclasses of the family when they are applied to polynomial systems is also performed (see [1] and [2]), showing their performance depending on the initial iterates.

Referencias

[1] A. Cordero, J. R. Torregrosa, F. Soleymani, Dynamical analysis of iterative methods for nonlinear systems or how to deal with the dimension?, Applied Mathematics and Computation 244 (2014) 398–412.

[2] R. C. Robinson, An Introduction to Dynamical Systems, Continous and Discrete. Americal Mathematical Society, Providence, 2012.

Joint work with Francisco I. Chicharro, Alicia Cordero and Juan R. Torregrosa.
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El plano de parámetros del método de Newton aplicado a polinomios de grado cuatro con una raíz doble

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Resumen. El estudio dinámico del método de Newton aplicado a polinomios de grado cuatro no ha sido abordado en su totalidad. Existen resultados parciales para algunas familias concretas, como por ejemplo polinomios simétricos $p(z) = z^4 + az^3 + bz^2 + az + 1$ con a y b reales [1] o polinomios de Bring-Jerrard $p(z) = z^4 - az + 1$, $a \in \mathbb{C}$ [2]. En este trabajo consideramos otro caso particular, en concreto polinomios de la forma

$$p(z) = (z-a)(z-b)(z-c)^2, \quad a, b, c \in \mathbb{C}. \tag{1}$$

Como el método de Newton aplicado a polinomios de la forma (1) tiene 2 puntos críticos libres, podemos dibujar su espacio de parámetros con 8 colores, con unas características y forma de estudio similar al realizado en [3] para el método de Chebyshev.

Referencias

[1] B. Campos, A. Garijo, X. Jarque y P. Vindel: Newton method for symmetric quartic polynomial. *Applied Mathematics and Computation* 290 (2016) 326–335.

[2] B. Campos, A. Garijo, X. Jarque y P. Vindel: Newton method on Bring-Jerrard polynomials. *Publ. Mat.* Volume Extra (2014) 81–109.

[3] J. M. Gutiérrez y J. L. Varona: A new parameter plane for Chebyshev's method. *Proceedings XXV CEDYA + XV CMA*, pp. 350–354, Cartagena, 2017.

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Majorizing sequences and iterative processes

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Resumen. Remember that a majorizing sequence is a scalar sequence that majorizes a sequence defined in a Banach space and its interest lies in deducing the convergence of the sequence in the Banach space from the convergence of the scalar sequence. Kantorovich was the first one in defining a majorizing sequence from his known majorant principle [1], that is based on an interpolation fitting of the conditions of convergence. Rheinboldt proposes in [2] the construction of majorizing sequences from the solution of difference equations. Both techniques have the severe limitation of the type of conditions of convergence that are required to the involved operator in the Banach space. In this work, we propose a different alternative to obtain majorizing sequences that also generalizes both techniques mentioned.

Referencias

[1] L. V. Kantorovich and G. P. Akilov, Functional analysis, Pergamon Press, Oxford, 1982.

[2] W. C. Rheinboldt, A unified convergence theory for a class of iterative processes, SIAM J. Numer. Anal., 5 (1968) 42–63.

Trabajo en colaboración con José Antonio Ezquerro.
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Ball convergence for a novel fourth order method for solving systems of equations

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Abstract. We present a local convergence analysis of a novel fourth order method in order to approximate a locally unique solution of a system of nonlinear equation $F(x) = 0$, where operator F is defined between two Banach spaces. In earlier studies, the fourth order of the method was established in [1], and in this study we expand the applicability of the method by means of using weaker hypotheses. A ball of convergence, error estimates on the distances involved and a uniqueness result are given using Lipschitz constants. Numerical examples are also presented in this study.

Referencias

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Serial and parallel iterative splitting methods: algorithms and applications

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Abstract. The properties of iterative splitting methods with serial versions have been analyzed since recent years, see [1]. We extend the iterative splitting methods to a class of parallel versions, which allow to reduce the computational time and keep the benefit of the higher accuracy with each iterative step. Parallel splitting methods are nowadays important to solve large problems, which can be splitted in subproblems and computed independently with the different processors. Such a flexibilisation with multisplitting methods allow to decompose large iterative splitting methods and recover the benefit of their underlying waveform-relaxation (WR) methods. We discuss the numerical convergence of the serial and parallel iterative splitting methods and present different numerical applications to validate the benefit of the parallel versions.

Referencias

[1] I. Farago and J. Geiser. *Iterative Operator-Splitting methods for Linear Problems.* International Journal of Computational Science and Engineering, 3 (4), 255–263, 2007.

Joint work with Jürgen Geiser, Ruhr University of Bochum (Germany) and José Luis Hueso, Universitat Politècnica de València.

On an efficient third order iterative scheme for solving a quadratic matrix equation

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Abstract. The quadratic matrix equation $X^2 - BX - C = 0$, in $n \times n$ matrices arises in numerous applications and is of intrinsic interest as one of the simplest nonlinear matrix equations. A natural contender for solving the quadratic matrix equation is the Newton method, which has been investigated for several authors, see for instance [1], [2], [3]. We show an efficient third order iterative scheme for solving the above quadratic matrix equation. Numerical experiments confirm the very good performance of this scheme.

Referencias

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[3] N. J. Higham and H. M. Kim, Solving a quadratic matrix equation By Newton's method with exact line searches, SIAM J. Matrix Anal. Appl., 23, (2001), 303–316.

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Iterative Methods for Approximating Generalized Inverse $A^{(2)}_{T,S}$ of a Complex Matrix A

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Abstract. The classical generalized inverses such as the Moore-Penrose inverse, the Drazin inverse, the group inverse, the Bott-Duffin inverse, and so on, are of special interest in matrix theory. They are extensively used in statistics, control theory, power systems, optimization, etc. Most of these generalized inverses are outer inverses of the form $A^{(2)}_{T,S}$ having the prescribed range T and null space S. Given a complex matrix $A \in \mathbb{C}^{m\times n}$, the unique matrix $X \in \mathbb{C}^{n\times m}$ such that $XAX = X$, $R(X) = T$ and $N(X) = S$ is known as the outer inverse or 2-inverse, denoted by $A^{(2)}_{T,S}$, of A with the prescribed range T and null space S. The aim of this paper is to present and analyze new high order iterative method for approximating the generalized inverse $A^{(2)}_{T,S}$ for a given matrix A. We also discuss how the new method may be applied for computing the inverse of nonsingular complex matrices. Some numerical tests are presented for confirming the theoretical results and for comparing our scheme with other known ones. All the examples are random matrices of big size.

Referencias

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Nuevos resultados para la aproximación rápida y certificada de la descomposición en valores singulares

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Resumen. En este trabajo, presentamos un algoritmo eficiente para la certificación de descomposiciones de valores singulares numéricos (SVD) en el caso general, es decir, cuando hay varios valores singulares. Nuestro algoritmo se basa en una iteración de tipo Newton, que también se puede utilizar para duplicar la precisión de un valor numérico aproximado de la solución.

Trabajo en colaboración con Joris van der Hoeven, Laboratoire LIX.